Math 1B Quiz 1 Version 1
Tue Jan 12, 2016

NAME YOU ASKED TO BE CALLED IN CLASS:

SCORE: 13 **/ 30 POINTS**

1. **NO CALCULATORS OR NOTES ALLOWED**
2. **SHOW PROPER CALCULUS-LEVEL WORK**
3. **SIMPLIFY ALL ANSWERS**

For this question, you may use the formulae for $\frac{d}{dx}\sinh x$, $\frac{d}{dx}\cosh x$ and/or $\frac{d}{dx}\tanh x$ without proving them. **SCORE:** $4\frac{1}{2}$ **/ 8 PTS**

If you need to use the formula for the derivative of any other hyperbolic function, you must prove it.

[a] **Without using the exponential formula for sech x**, prove the formula for $\frac{d}{dx}\operatorname{sech} x$.

$\frac{d}{dx}\operatorname{sech} x$

$= \frac{d}{dx}\frac{1}{\cosh x}$

$= -\frac{1}{\cosh^2 x}\cdot \sinh x$ (2)

$= -\frac{\sinh x}{\cosh^2 x}$

[b] **Without using the logarithmic formula for $\tanh^{-1} x$**, prove the formula for $\frac{d}{dx}\tanh^{-1} x$.

$y = \tanh^{-1} x$.

$x = \tanh y$ (1)

$dx = dy \cdot \frac{1}{\cosh^2 y}$

$\frac{dy}{dx} = \cosh^2 y$

$\frac{dy}{dx} = \frac{1}{1-\tanh^2 y}$.

$\frac{dy}{dx} = \frac{1}{1-x^2}$

$\therefore \frac{d}{dx}\tanh^{-1} x = \frac{1}{1-x^2}$ $(1\frac{1}{2})$

$(\tanh x)' = \left(\frac{\sinh x}{\cosh x}\right)' = \frac{1}{\cosh x}\sinh x$

Find $\lim_{x\to\infty}\coth x$ **algebraically**.

SCORE: 0 **/ 3 PTS**

$\lim_{x\to\infty}\coth x$

$\lim_{x\to\infty}\frac{\cosh x}{\sinh x}$

$\lim_{x\to\infty}\frac{\frac{e^x+e^{-x}}{2}}{\frac{e^x-e^{-x}}{2}}$

$= \lim_{x\to\infty}\frac{e^x+e^{-x}}{e^x-e^{-x}}$

$\left(\lim_{x\to\infty}\coth x\right)^2$

$= \lim_{x\to\infty}\coth^2 x$

$= \lim_{x\to\infty}\frac{e^{2x}-e^{-2x}+2}{e^{2x}-e^{-2x}-2}$

$= \lim_{x\to\infty}\left(1+\frac{4}{e^{2x}-e^{-2x}-2}\right)$

$= 1$

$\therefore \lim_{x\to\infty}\coth x = \pm 1$

If $\coth x = -5$, find $\sinh x$.

SCORE: $1\frac{1}{2}$ / 4 PTS

$\coth^2 x = 25$, $\tanh = -\frac{1}{5}$, $(\tanh)^2 = \frac{1}{25}$

$\sinh^2 x (\coth^2 x - 1) = 1$

$\therefore \sinh^2 x = \frac{1}{24}$

$\sinh x = \pm \frac{1}{2\sqrt{6}}$

$\because \coth x = -5 < 0$

$\therefore \frac{\cosh x}{\sinh x} < 0$

$\because \cosh x = \frac{e^x + e^{-x}}{2} > 0$ $\therefore \sinh x < 0$ (1/2)

$\therefore \sinh x = -\frac{1}{2\sqrt{6}} = -\frac{\sqrt{6}}{12}$ (1/2)

Prove the logarithmic formula for $\sinh^{-1} x$ given in your textbook.

NOTE: This is NOT a question about derivatives.

SCORE: 3 / 5 PTS

$y = \sinh^{-1} x$

$x = \sinh y$

$x = \frac{e^y - e^{-y}}{2}$ (1)

$2x = e^y - e^{-y}$

let $e^y = t$ (1/2)

$e^{-y} = \frac{1}{t}$

$t - \frac{1}{t} = 2x$

$t^2 - 1 = 2tx$

$t^2 - 2tx = 1$

$(t-x)^2 = 1 + x^2$

$t - x = \pm\sqrt{1+x^2}$

$t = x \pm \sqrt{1+x^2}$ (1/2)

$\because e^y > 0$ $\therefore t = x + \sqrt{1+x^2}$ (1/2)

$\therefore e^y = x + \sqrt{1+x^2}$

$\therefore y = \ln(x + \sqrt{1+x^2})$ (1/2)

$\therefore \sinh^{-1} x = \ln(x + \sqrt{1+x^2})$.

Find $\frac{d}{dx} x^2 \cosh^{-1}(x^5)$. Simplify your final answer as a single fraction.

SCORE: 2 / 4 PTS

You may use the derivatives of any hyperbolic or inverse hyperbolic functions from your textbook without proving them.

$\frac{d}{dx} x^2 \cdot \cosh^{-1}(x^5)$

$= 2x \cosh^{-1}(x^5) + x^2 \cdot 5x^4 \cdot \frac{1}{\sqrt{x^2-1}}$

$= 2x \cosh^{-1}(x^5) + \frac{5x^6}{\sqrt{x^2-1}}$ (1)

(1)

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[MULTIPLE CHOICE] Write the letter of the correct answers in the spaces below.

ANSWERS: [1] f [2] d [3] e [4] e [5] b [6] e

[1] If you score 110 points on Midterm 1, 130 points on Midterm 2 and 135 points on Midterm 3, which midterm score(s) will be changed, and to what value ?
(HINT: You are encouraged to start studying regularly early in the quarter.)

[a] Midterm 1's score will be changed to 135 (the highest midterm score)
[b] Midterm 1's score will be changed to (110 + 130 + 135) ÷ 3 = 125 (the average of all midterm scores)
[c] Midterm 1's score will be changed to (110 + 130) ÷ 2 = 120 (the average of Midterm 1's and Midterm 2's scores)
[d] Midterm 1's score will be changed to (110 + 135) ÷ 2 = 122.5 (the average of Midterm 1's and the highest midterm's scores)
[e] Midterm 1's score will be changed to (110 + 135) ÷ 2 = 122.5 and Midterm 2's score will be changed to (130 + 135) ÷ 2 = 132.5 (the average of each midterm's and the highest midterm's score)
[f] no midterm scores will be changed

[2] If you score 145 points on Midterm 1, 125 points on Midterm 2 and 150 points on Midterm 3, which midterm score(s) will be changed, and to what value ?

[a] Midterm 2's score will be changed to 150 (the highest midterm score)
[b] Midterm 2's score will be changed to (125 + 145 + 150) ÷ 3 = 140 (the average of all midterm scores)
[c] Midterm 2's score will be changed to (125 + 145) ÷ 2 = 135 (the average of Midterm 2's and Midterm 1's scores)
[d] Midterm 2's score will be changed to (125 + 150) ÷ 2 = 137.5 (the average of Midterm 2's and the highest midterm's scores)
[e] Midterm 1's score will be changed to (145 + 150) ÷ 2 = 147.5 and Midterm 2's score will be changed to (125 + 150) ÷ 2 = 137.5 (the average of each midterm's and the highest midterm's score)
[f] no midterm scores will be changed

[3] Which statement below regarding attendance is **false** ?

[a] Whenever you come into class (whether on time or late), you should sign in on the attendance spreadsheet right away.
[b] Arriving late on a quiz or midterm day will not be counted as late.
[c] Unexcused early departures are considered absences.
[d] If you have perfect attendance and classroom behavior for the first 7 weeks, and do not show up again after that, you will receive an F for the course.
[e] Attendance policies will not apply to you if you score more than 80% on every midterm.

[4] How much of your learning does the instructor believe comes from your daily reading and homework ?

[a] reading = 10%, homework = 30% ➡ combined = 40%
[b] reading = 15%, homework = 35% ➡ combined = 50%
[c] reading = 15%, homework = 45% ➡ combined = 60%
[d] reading = 20%, homework = 50% ➡ combined = 70%
[e] reading = 25%, homework = 55% ➡ combined = 80%

[5] Proper use of the textbook for this class includes

[a] understanding all the terminology used in the book
[b] working out the given examples yourself and checking that you are able to get the same results as the book
[c] reading the sections of the textbook before the corresponding lecture
[d] all of the previous answers [a], [b] and [c]
[e] some, but not all, of the previous answers [a], [b] and [c]

[6] Which statement below regarding tests (quizzes, midterms, final exam) is **false** ?

[a] If you continue writing on your test after the stated ending time, you will receive a 0 for that test.
[b] There are no make-ups for missed quizzes.
[c] The instructor expects you to be able to identify and execute solutions on midterms more quickly than on quizzes because you should have had much more practice.
[d] If your tablet, phone, computer etc. makes an audible noise during a test, you will lose 10% of all points available on that test.
[e] If you cannot make the scheduled final exam time for any reason, your final exam can be rescheduled.